Trigonometry **Special Topic: Review of Logarithms**

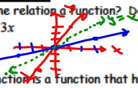
Inverse of a Function

Vertical Line Test—Used to determine if a relation is a function

Horizontal Line Test—Used to determine if a function has an inverse

Example: Is the relation a function? Does it have an inverse? If so, find it.

$y = 3x$



D R

x	y
0	0
1	3
-1	-3

$y = 3x$
$x = 3y$
$\frac{x}{3} = y$

$\frac{x}{3} = f^{-1}(x)$

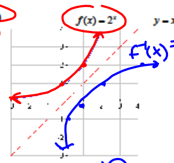
D R

x	y
0	0
3	1
-3	-1

An exponential function is a function that has a variable as an exponent.

$D: (-\infty, \infty)$
$R: (0, \infty)$

$f(x) = 2^x$ $y = x$



D R

x	y
1	2
0	1
2	4
-1	$\frac{1}{2}$

$f^{-1}(x) = \log_2 x$

x	y
2	1
1	0
4	2
$\frac{1}{2}$	-1

Inverse of Exponent

$y = 2^x$
$x = 2^y$
$\log_2 x = y$

Request "2 to what power is x"

A logarithm is an exponent.

$y = 2^x$
$x = 2^y$
$\log_2 x = y$

Logarithmic Form: $\log_a x = y$

Exponential Form: $x = a^y$

Exponential Form	Logarithmic Form
$3^x = 7$	$\log_3 7 = x$
$x^5 = 12$	$\log_x 12 = 5$
$4^x = 6$	$\log_4 6 = x$
$11^5 = x$	$\log_{11} x = 5$

log common log (base 10)

ln natural log (base e) $\log_e$

Properties of Logarithms

- $\log_a x$ is defined if and only if x is positive $\ln e^x$
- $\log_a 1 = 0$ because $a^0 = 1$ $\ln e^1 = 0$ $e^0 = 1$
- $\log_a a = 1$ because $a^1 = a$ $\ln e^e = 1$ $e^1 = e$
- $\log_a a^x = x$ *and* $a^{\log_a x} = x$ (inverse property)
- If $\log_a x = \log_a y$ then x = y $\ln x = \ln y$

Laws of Exponents

1. $a^m \cdot a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$; $m > n$
3. $(a^m)^n = a^{mn}$

Product Rule $\log_b MN = \log_b M + \log_b N$

Quotient Rule $\log_b \frac{M}{N} = \log_b M - \log_b N$

Power Rule $\log_b M^k = k \log_b M$

Other Thoughts:

An exponential function is a function that has a variable as an exponent.

A logarithm of a number in a certain base is an exponent.

To solve problems in logarithmic form, you sometimes have to go into exponential form and visa-versa. (Two Way Street)

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Trigonometry **Logarithm Review Worksheet** **Name:**________________

Write the logarithmic equation in exponential form.

1. $\log_4 64 = 3$ $4^3 = 64$
2. $\log_3 81 = 4$
3. $\log_7 \frac{1}{49} = -2$
4. $\log_{32} 4 = \frac{2}{5}$
5. $\ln 1 = 0$ $e^0 = 1$

Write the exponential equation in logarithmic form.

6. $5^3 = 125$
7. $81^{\frac{1}{4}} = 3$
8. $9^{\frac{3}{2}} = 27$
9. $6^{-2} = \frac{1}{36}$
10. $e^0 = 1$ $\ln 1 = 0$
11. $e^x = 4$

Evaluate the expression without using a calculator.

12. $\log_2 16$ $2^x = 16$ $2^x = (2)^4$ $x = 4$
13. $\log_{27} 9$
14. $\log_{16} \frac{1}{4}$
15. $\ln e^3 - \ln e^7$ $3 \ln e - 7 \ln e$ $3 - 7$ -4
16. $\log_{10} 0.01$
17. $\ln e^{\frac{2}{3}}$

Solve the equation for x.

18. $\log_7 x = \log_7 9$ $x = 9$
19. $\log_6 36 = x$
20. $\log_2 \left(\frac{1}{2}\right) = x$ $2^x = \frac{1}{2}$ $2^x = 2^{-1}$ $x = -1$
21. $\ln e^8 = x$ $e^x = e^8$ $x = 8$
22. $2^x = 8$
23. $4^x = 8$
24. $5^{x+1} = \frac{1}{125}$
25. $2^x = 5$ $\log 2^x = \log 5$ $x(\log 2) = \log 5$ $x = \frac{\log 5}{\log 2} \approx 2.322$
26. $\log_x 25 = 2$
27. $e^{-x} = 3$ $\ln e^{-x} = \ln 3$ $-x \ln e = \ln 3$ $x = -\ln 3$ ≈ -1.099
28. $\ln \frac{1}{x} = 4$
29. $7^{3x} = 12$
30. $e^{3x} = 27$
31. $\sqrt{e^x} = 3$
32. $\ln(x-3) = 2$ $e^2 = x - 3$ $e^2 + 3 = x$

Evaluate the logarithm using the change of base formula. $\boxed{\log_a b = \frac{\log b}{\log a}}$

33. $\log_3 7 = x$ $3^x = 7$ $\log 3^x = \log 7$ $x \log 3 = \log 7$ $x = \frac{\log 7}{\log 3}$ $x \approx 1.771$
34. $\log_{\frac{1}{2}} 4$
35. $\log_9 0.8$
36. $\log_{20} 135$

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Use the properties of logarithms to write the expression as a sum, difference, and/or constant multiple of logarithms (assume all variables are positive).

37. $\log_{10} 5x$

$\log_{10} 5 + \log_{10} x$

38. $\log_{10} 10w$

$\log_{10} 10 + \log_{10} w$

$1 + \log w$

39. $\log_{10} \frac{5}{x}$

40. $\log_{10} \frac{xy}{2}$

$\log_{10} x + \log_{10} y - \log_{10} 2$

41. $\log_8 x^4$

42. $\log_6 \left(\frac{z}{w}\right)^{-6}$

43. $\ln \sqrt{w}$

44. $\ln \sqrt[3]{xy}$

45. $\ln \frac{x(x+1)^2}{y}$

46. $\log \frac{x}{y\sqrt{z}}$

Write the expression as the logarithm of a single quantity.

47. $\ln x + \ln 4$

$\ln 4x$

48. $\log_4 z - \log_4 y$

$\log_4 \frac{z}{y}$

49. $2\log_2 (w+3)$

$\log_2 (w+3)^2$

$b^{\frac{P}{I}}$

$\sqrt[I]{b^P}$

50. $\frac{1}{3}\log_3 7x$

$\log_3 (7x)^{\frac{1}{3}}$

$\log_3 \sqrt[3]{7x}$

51. $-6\log_5 2x$

52. $\ln x - 3\ln(x+1)$

53. $2\ln 8 + 5\ln w$

54. $\frac{1}{2}\log 64 - \log x$

55. $\log x - \log y - \log z$

56. $3\ln x - 2\ln y + 4\ln z$

57. $\ln 10 - 3\ln x + \ln(x+1)$

$\ln 10 - \ln x^3 + \ln(x+1)$

$\ln \frac{10}{x^3} + \ln(x+1)$

$\ln \frac{10(x+1)}{x^3}$

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