Calculus 7.2 Volume: The Disk Method

Objective: Find the volume of a solid of revolution using the disk method.
Find the volume of a solid of revolution using the washer method.

Solid of Revolution A region of a plane that is revolved about a line. The line is called the axis of revolution.

V = (area of disk)(width of disk) = $\pi R^2 w$ (p.456)

$\Delta V = \pi R^2 \Delta x$

$V_{solid} = \sum_{i=1}^{n} \pi [R(x_i)]^2 \Delta x$

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Better approximations occur as $n \to \infty$. $V = \lim_{n \to \infty} \pi \sum_{i=1}^{n} [R(x_i)]^2 \Delta x$

Known Formula	Representative Element	New Integration Formula
Volume of Disk		Solid of Revolution
$V = \pi R^2 w$	$\Delta V = \pi [R(x_i)]^2 \Delta x$	$V = \pi \int_a^b [R(x)]^2 \, dx$

The Disk Method (p.457)

To find the volume of a solid of revolution with the disk method, use one of the following:

Horizontal Axis of Revolution (y=)

$V = \pi \int_a^b [R(x)]^2 \, dx$

Vertical Axis of Revolution (x=)

$V = \pi \int_c^d [R(y)]^2 \, dy$

Example 1 Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = \sqrt{\cos x}$ and the x-axis $(-\frac{\pi}{2} \le x \le \frac{\pi}{2})$ about the x-axis.

Solid - Disk

$V = \pi \int_{-\pi/2}^{\pi/2} [\sqrt{\cos x}]^2 \, dx$

$= \pi [\sin x]_{-\pi/2}^{\pi/2}$

$= \pi (1 - (-1))$

$= 2\pi$

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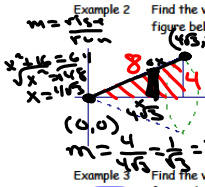
Example 2 Find the volume of the solid formed by revolving the region bounded by the figure below about the x-axis.

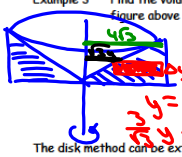


Solid - Disk

$V = \pi \int_0^{4\sqrt{3}} \left[\frac{\sqrt{3}}{3} x\right]^2 dx$

$= \pi \int_0^{4\sqrt{3}} \frac{1}{3} x^2 \, dx$

Example 3 Find the volume of the solid formed by revolving the region bounded by the figure above about the y-axis.



Hollow Washer

$V = \pi \int_0^4 [4\sqrt{3}]^2 - [\sqrt{3} y]^2 \, dy$

$y = \frac{\sqrt{3}}{3} x$

$\sqrt{3} y = x$ $x = \sqrt{3} y$

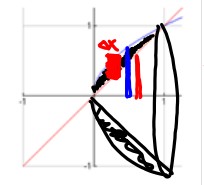
The disk method can be extended to cover solids of revolution with holes by replacing the representative disk with a representative washer. The washer is formed by revolving a rectangle about an axis. If r and R are the inner and outer radius of the washer and w is the width of the washer, the volume is given by

$Volume of Washer = \pi (R^2 - r^2) w$

$V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) \, dx$ washer method

special case: solids of revolution with holes

Example 4 Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x$ about the x-axis $(0 \le x \le 1)$.
(concavity, carved out piece)



Washer

$V = \pi \int_0^1 [\sqrt{x}]^2 - [x]^2 \, dx$

$= \pi \int_0^1 x - x^2 \, dx$

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