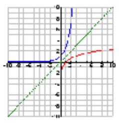
Calculus 5.4 Exponential Functions: Differentiation and Integration

Calculate the inverse of each function.

$f(x) = e^x$ $\qquad$ $g(x) = \ln x$



Solve for x accurate to three decimal places.

$\ln e = 1$

1. $\ln 7 = e^{x+1}$
   $\ln 7 = \ln(e)^{x+1}$
   $\ln 7 = (x+1)\cancel{\ln e}$
   $\ln 7 - 1 = x$

2. $e^{\ln 5} = \ln x$
   $\ln e^{\ln 5} = \ln x$
   $(\ln 5)(\cancel{\ln e}) = \ln x$
   $x = 5$

3. $\ln(2x-3) = 5$
   $e^5 = 2x - 3$
   $\dfrac{e^5 + 3}{2} = x$

Derivative of the Natural Exponential Function

Let u be a differentiable function of x.

1. $\dfrac{d}{dx}\left[e^x\right] = e^x$ $\qquad$ 2. $\dfrac{d}{dx}\left[e^u\right] = e^u\dfrac{du}{dx}$ or $e^u u'$

Find the derivative of the function.

1. $f(x) = e^{2x-1}$ $\quad u = 2x-1$, $u' = 2$
   $f'(x) = u'e^u$
   $= 2e^{2x-1}$

2. $y = e^{-\frac{3}{x}} = e^{-3x^{-1}}$ $\quad u = -3x^{-1}$, $u' = 3x^{-2} = \dfrac{3}{x^2}$
   $y' = u'e^u$
   $y' = \dfrac{3}{x^2} \cdot e^{\frac{-3}{x}} = \dfrac{3}{x^2 e^{\frac{3}{x}}}$

$y = e^{-x}$ $\quad u = -x$, $u' = -1$
$y' = u'e^u$
$= -1e^{-x}$

3. $y = \dfrac{e^x - e^{-x}}{2}$
   $y = \frac{1}{2}(e^x - e^{-x})$
   $y' = \frac{1}{2}(e^x + (+e^{-x}))$
   $= \frac{1}{2}\left(e^x + \frac{1}{e^x}\right)$
   $= \dfrac{e^{2x}+1}{2e^x}$

4. $f(x) = \ln\left(\dfrac{e^x + e^{-x}}{2}\right)$
   $f(x) = \ln(e^x + e^{-x}) - \ln 2$
   $f'(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} - 0$
   $= \dfrac{\left(e^x - \frac{1}{e^x}\right)e^x}{\left(e^x + \frac{1}{e^x}\right)e^x} = \dfrac{e^{2x}-1}{e^{2x}+1}$

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Use implicit differentiation to find $\dfrac{dy}{dx}$.

5. $\dfrac{d}{dx}\left[xe^y - 10x + 3y\right] = \dfrac{d}{dx}\left[0\right]$
   $x(y'e^y) + e^y(1) - 10 + 3y' = 0$
   $y'(xe^y + 3) = 10 - e^y$
   $y' = \dfrac{10 - e^y}{xe^y + 3}$

6. Find the extrema and the points of inflection (if any exist) of the function.
   $f(x) = 1 + (2+x)e^{-x}$
   $f'(x) = 0 + (2+x)(-1e^{-x}) + e^{-x}(1)$
   $0 = -2e^{-x} - xe^{-x} + e^{-x}$
   $0 = \boxed{-e^{-x} - xe^{-x}}$
   $0 = e^{-x}(-1-x)$
   $e^{-x} = 0$ NP $\quad -1 - x = 0$
   $-1 = x$ CN

	$x<-1$	$x=-1$	$x>-1$
$f'(x)$	+	0	−

Rel Max $(-1, 1+e)$

$f'(x) = -e^{-x} - xe^{-x}$
$f''(x) = 1e^{-x} - [(x)(-e^{-x}) + e^{-x}(1)]$
$= e^{-x} + xe^{-x} - e^{-x}$
$0 = xe^{-x}$
$x = 0$

	$x<0$	$x=0$	$x>0$
$f''(x)$	−	0	+
	CD		CU

POI $(0, 3)$

Integration Rules for Exponential Functions

Let u be a differentiable function of x.

1. $\int e^x\,dx = e^x + C$ $\qquad$ 2. $\int e^u\,du = e^u + C$

Find or evaluate the integral.

1. $\int e^{3x+1}\,dx$ $\qquad$ 2. $\int 5xe^{-x^2}\,dx$

3. $\int \dfrac{e^{\frac{1}{x}}}{x^2}\,dx$ $\qquad$ 4. $\int_0^1 \dfrac{e^x}{1+e^x}\,dx$

5. Find the area of the region bounded by the graphs of the equations.
   $y = xe^{\frac{-x^2}{4}}, y = 0, x = 0, x = \sqrt{6}$

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