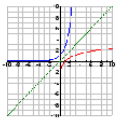
Calculus **5.4 Exponential Functions: Differentiation and Integration**

Calculate the inverse of each function.

$f(x) = e^x$ $g(x) = \ln x$



Solve for x accurate to three decimal places.

1. $7 = e^{x+1}$ 2. $e^{\ln 5} = x$ 3. $\ln(2x-3) = 5$

Derivative of the Natural Exponential Function

Let u be a differentiable function of x.

1. $\frac{d}{dx}[e^x] = e^x$ 2. $\frac{d}{dx}[e^u] = e^u \frac{du}{dx}$ or $e^u u'$

Find the derivative of the function.

1. $f(x) = e^{2x-1}$ 2. $y = e^{-\frac{3}{x}}$

3. $y = \frac{e^x - e^{-x}}{2}$ 4. $f(x) = \ln\left(\frac{e^x + e^{-x}}{2}\right)$

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Use implicit differentiation to find $\frac{dy}{dx}$.

5. $xe^y - 10x + 3y = 0$

6. Find the extrema and the points of inflection (if any exist) of the function.
$f(x) = 1 + (2+x)e^{-x}$

Integration Rules for Exponential Functions

Let u be a differentiable function of x.

1. $\int e^x dx = e^x + C$ 2. $\int e^u du = e^u + C$

Find or evaluate the integral.

1. $\frac{1}{3}\int e^{3x+1} dx$ $u = 3x+1$, $du = 3dx$ 2. $\int 5xe^{-x^2} dx$

$\frac{1}{3}e^{3x+1} + C$ $\frac{5}{-2}\int e^{-x^2}(-2)x\,dx$ $u = -x^2$, $du = -2x\,dx$

$-\frac{5}{2}e^{-x^2} + C = -\frac{5}{2e^{x^2}} + C$

3. $\int \frac{e^{\frac{1}{x}}}{x^2} dx$ $u = x^{-1}$, $du = -1x^{-2}dx$ 4. $\int_0^1 \frac{e^x}{(1+e^x)^2} dx$ $u = 1+e^x$, $u' = e^x$

$-\int e^{x^{-1}} \cdot \frac{1}{x^2} dx$ $\ln|1+e^x|_0^1$

$-e^{x^{-1}} + C$ $\ln|1+e| - \ln|1+1| = \ln\left(\frac{1+e}{2}\right)$

$-e^{\frac{1}{x}} + C$

5. Find the area of the region bounded by the graphs of the equations.
$y = xe^{-\frac{x^2}{4}}, y = 0, x = 0, x = \sqrt{6}$

$u = -\frac{x^2}{4}$, $du = -\frac{1}{2}x\,dx$

$\int_0^{\sqrt{6}} x e^{-\frac{x^2}{4}} dx$ $-2\int_0^{\sqrt{6}} e^{-\frac{x^2}{4}} \cdot -\frac{1}{2}x\,dx$

$-2e^{-\frac{x^2}{4}}\Big|_0^{\sqrt{6}}$ $= -2e^{-\frac{6}{4}} + (+2)$

$= -\frac{2}{e^{\frac{3}{2}}} + 2 = -\frac{2+2e^{\frac{3}{2}}}{e^{\frac{3}{2}}}$

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